

Black Hole-Neutron Star Mergers: Disk Mass Predictions

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Determining the final result of black hole-neutron star mergers, and in particular the amount of matter remaining outside the black hole at late times, has been one of the main motivations behind the numerical simulation of these systems. Black hole-neutron star binaries are amongst the most likely progenitors of short gamma-ray bursts — as long as they result in the formation of massive (at least $\sim 0.1M_\odot$) accretion disks around the black hole. Whether this actually happens strongly depends on the physical characteristics of the system, and in particular on the mass ratio, the spin of the black hole, and the radius of the neutron star. We present here a simple two-parameter model, fitted to existing numerical results, for the determination of the mass remaining outside the black hole a few milliseconds after a black hole-neutron star merger. This model predicts the remnant mass within a few percents of the mass of the neutron star, at least for remnant masses up to 20% of the neutron star mass. Results across the range of parameters deemed to be the most likely astrophysically are presented here. We find that, for $10M_\odot$ black holes, massive disks are only possible for fairly large neutron stars ($R_{\text{NS}} \gtrsim 12\text{km}$), or quasi-extremal black hole spins ($a_{\text{BH}}/M_{\text{BH}} \gtrsim 0.9$). We also use our model to discuss how the equation of state of the neutron star affects the final remnant, and the strong influence that this can have on the rate of short gamma-ray bursts produced by black hole-neutron star mergers.

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I. INTRODUCTION

The potential of black hole-neutron star (BHNS) mergers as progenitors of short gamma-ray bursts (SGRBs) and their importance as sources of gravitational waves detectable by ground-based interferometers such as Advanced LIGO and VIRGO [1–3] have driven most recent studies of these systems. Gamma-ray bursts, in particular, are a likely result if the neutron star is tidally disrupted, and the final outcome of the merger is a massive accretion disk around the black hole (see [4] and references therein).

Numerical simulations have taught us that BHNS mergers can be divided into two broad categories: either tidal effects are strong enough that the neutron star is disrupted before reaching the innermost stable circular orbit (ISCO) of the black hole, or the neutron star plunges into the hole before tidal disruption occurs. In the first case, some material from the disrupted star remains outside the black hole for long periods of time ($\sim 0.1\text{--}1\text{s}$) in the form of an accretion disk, a tidal tail, and/or unbound ejecta. In the second case, however, the entire neutron star is rapidly accreted onto the black hole. To first order, the most important parameters determining the outcome of a BHNS merger are the mass ratio of the binary [5–7], the spin magnitude of the black hole [6–8], its orientation [8], and the size of the neutron star [5, 9, 10]. The formation of massive accretion disks is more likely to occur for black holes of low mass (at least down to mass ratios $M_{\text{BH}}/M_{\text{NS}} \sim 3$) and high spins, and for large neutron stars.

Studying these mergers is a complex problem, and accurate results can only be obtained through numerical simulations in a general relativistic framework: results using approximate treatments of gravity can lead to qualitative differences in the dynamics of the merger, and large errors in the mass of the final accretion disk or of any unbound material. Unfortunately, general relativistic simulations are computationally expensive, and only ~ 50 BHNS mergers have been studied so far (see [11, 12] for reviews of these results). Additionally,

a majority of these simulations considered binaries with mass ratios $M_{\text{BH}}/M_{\text{NS}} \sim 2\text{--}3$, while population synthesis models indicate that mass ratios $M_{\text{BH}}/M_{\text{NS}} \geq 5$ are astrophysically more likely [13, 14]. Existing general relativistic simulations are also fairly limited in the physical effects considered: only a few include magnetic fields [15, 16] or nuclear theory-based equations of state [10], and none have considered neutrino emission (although neutrinos have been included in simulations of neutron star-neutron star mergers [17]). Magnetic fields and neutrinos do not affect significantly the disruption of the star, or the amount of matter remaining outside the black hole after merger, but they are critical to the evolution of the post-merger remnant, and to the modelling of electromagnetic and neutrino counterparts to the gravitational wave signal emitted by black hole-neutron star mergers.

Given the size of the parameter space to explore and the cost of numerical simulations, obtaining accurate predictions for the final state of the system for all possible configurations is only feasible through the construction of a model which effectively interpolates between known numerical results. Such a model can also be of great help to determine which binary parameters should be used in numerical simulations in order to study a specific physical effect (e.g. massive disks) without having to run many different configurations. Taniguchi et al. [18] obtained a first estimate of the limit between configurations in which no disk is formed and those for which some matter remains at late time by studying quasi-equilibrium configurations of non-spinning binaries. More recently, Pannarale et al. [19] computed estimates for the mass remaining outside the black hole at late times through the use of a toy-model studying the tidal forces acting on the neutron star, represented by a tri-axial ellipsoid. However, their model was fitted to numerical simulations which underestimated the remnant masses, and at a time when general relativistic results were only available at low mass ratios $M_{\text{BH}}/M_{\text{NS}} \sim 2\text{--}3$. The qualitative dependence of the remnant mass in the parameters of the binary is captured by their model, but the quantitative

results do not match more recent simulations, particularly at higher mass ratios [7].

In this paper we show that much simpler models comparing the estimated separation at which tidal disruption of the neutron star occurs (d_{tidal}) and the radius of the ISCO (R_{ISCO}) can accurately predict the mass remaining outside the black hole at late times. We fit two such models (with different approximations for d_{tidal}) to a set of 26 recent numerical simulations covering mass ratios in the range $M_{\text{BH}}/M_{\text{NS}} = 3 - 7$, black hole spins up to 0.9 and neutron star radii $R_{\text{NS}} \approx 11 - 16$ km. The case of black hole spins misaligned with the orbital angular momentum is not considered here, and we limit ourselves to low eccentricity orbits (high eccentricities only occur when the binary is formed through dynamical capture, e.g. in nuclear or globular clusters). Both models match the simulation results within their expected numerical errors, a few percents of the original mass of the neutron star. In Sec. II, we describe the models used, and their physical inspiration. Sec. III summarizes the numerical results used to calibrate the models, while Sec. IV gives the best-fit parameters, and discuss the quality of the fits. Finally, in Sec. V, we show predictions of these models across the entire parameter space. We also discuss their strong dependence in the size of the neutron star, and potential implications for the rate of short gamma-ray bursts originating from BHNS mergers.

II. TIDAL DISRUPTION MODELS

The models used here to estimate the mass remaining outside the black hole at late times are based on a comparison between the binary separation at which tidal forces become strong enough to disrupt the star, d_{tidal} , and the radius of the innermost stable circular orbit R_{ISCO} . Intuitively, if $d_{\text{tidal}} \lesssim R_{\text{ISCO}}$, the neutron star will plunge directly into the black hole and no mass will remain outside the hole after merger. On the other hand, if $d_{\text{tidal}} \gtrsim R_{\text{ISCO}}$, the star will be disrupted. Some disrupted material will then form an accretion disk, some will be ejected in a tidal tail and fall back on the disk over timescales long with respect to the duration of the merger (which typically lasts a few milliseconds), and some might be unbound.

The separation d_{tidal} at which tidal disruption occurs can be estimated in Newtonian theory by balancing the gravitational acceleration due to the star with the tidal acceleration due to the black hole:

$$\frac{M_{\text{NS}}}{R_{\text{NS}}^2} \sim \frac{3M_{\text{BH}}}{d_{\text{tidal}}^3} R_{\text{NS}} \quad (1)$$

$$d_{\text{tidal}} \sim R_{\text{NS}} \left(\frac{3M_{\text{BH}}}{M_{\text{NS}}} \right)^{1/3}, \quad (2)$$

where R_{NS} is the radius of the neutron star, M_{NS} and M_{BH} are the masses of the compact objects, and we work in units in which $G = c = 1$. In general relativity, these quantities are not uniquely defined. In practice we will use the radius of the star in Schwarzschild coordinates and the ADM mass of the compact objects, all measured at infinite separation.

As for the radius of the ISCO, it is given by [20]

$$\begin{aligned} Z_1 &= 1 + (1 - \chi_{\text{BH}}^2)^{1/3} \left[(1 + \chi_{\text{BH}})^{1/3} + (1 - \chi_{\text{BH}})^{1/3} \right] \\ Z_2 &= \sqrt{3\chi_{\text{BH}}^2 + Z_1^2} \\ \frac{R_{\text{ISCO}}}{M_{\text{BH}}} &= 3 + Z_2 - \text{sign}(\chi_{\text{BH}}) \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)} \end{aligned} \quad (3)$$

where $\chi_{\text{BH}} = a_{\text{BH}}/M_{\text{BH}}$ is the dimensionless spin parameter of the black hole.

To construct a model for the fraction of the baryon mass of the star remaining outside the black hole at late times, we assume that this mass is entirely determined by the relative location of R_{ISCO} and d_{tidal} , in units of the neutron star radius. A first guess for the remnant mass M_{model}^0 is then the linear model:

$$\frac{M_{\text{model}}^0}{M_{\text{NS}}^b} = \alpha^0 \frac{d_{\text{tidal}}}{R_{\text{NS}}} - \beta^0 \frac{R_{\text{ISCO}}}{R_{\text{NS}}} + \gamma_0, \quad (4)$$

where α^0 , β^0 and γ_0 are the free parameters of the model, and M_{NS}^b is the baryon mass of the neutron star. However, this simple prescription fits the numerical data rather poorly. In particular, it strongly underestimates the impact of the neutron star compactness $C_{\text{NS}} = M_{\text{NS}}/R_{\text{NS}}$ on the result. This problem is not overly surprising: d_{tidal} was derived in Newtonian gravity, but applied to compact objects. In particular, it predicts a finite radius for tidal disruption even if we replace the neutron star by a non-spinning black hole (for which $C = 0.5$). To improve the model, we use instead a corrected estimate of the distance for tidal disruption, in which compact objects are more strongly bound:

$$\tilde{d}_{\text{tidal}} = d_{\text{tidal}}(1 - 2C_{\text{NS}}). \quad (5)$$

This leads to the following model for the mass remaining outside the black hole at late times, $M_{\text{model}}^{\text{rem}}$:

$$\frac{M_{\text{model}}^{\text{rem}}}{M_{\text{NS}}^b} = \alpha (3q)^{1/3} (1 - 2C_{\text{NS}}) - \beta \frac{R_{\text{ISCO}}}{R_{\text{NS}}}, \quad (6)$$

with $q = \frac{M_{\text{BH}}}{M_{\text{NS}}}$. We could have added a constant term γ as in Eq. 4, but find that this does not improve the quality of the fit. At the current level of accuracy of numerical simulations, we will show in Sec. IV that this simple model is in agreement with known results.

A potential improvement on the model described by Eq. 6 is to compute the tidal effects from the Kerr metric instead of the Newtonian formula. Fishbone [21] obtained analytical results for these effects. Using his results leads to a correction to the value of the separation at which tidal disruption occurs: $\xi_{\text{tidal}} = d_{\text{tidal}}/R_{\text{NS}}$ is then solution of the implicit equation

$$\frac{M_{\text{NS}} \xi_{\text{tidal}}^3}{M_{\text{BH}}} = \frac{3(\xi_{\text{tidal}}^2 - 2\kappa \xi_{\text{tidal}} + \chi_{\text{BH}}^2 \kappa^2)}{\xi_{\text{tidal}}^2 - 3\kappa \xi_{\text{tidal}} + 2\chi_{\text{BH}} \sqrt{\kappa^3 \xi_{\text{tidal}}}} \quad (7)$$

with $\kappa = M_{\text{BH}}/R_{\text{NS}}$. We can then write the corrected model

$$\frac{\tilde{M}_{\text{model}}^{\text{rem}}}{M_{\text{NS}}^b} = \tilde{\alpha} \xi_{\text{tidal}} (1 - 2C_{\text{NS}}) - \tilde{\beta} \frac{R_{\text{ISCO}}}{R_{\text{NS}}}. \quad (8)$$

In practice, $\tilde{M}_{\text{model}}^{\text{rem}}$ gives results consistent with the simpler model $M_{\text{model}}^{\text{rem}}$.

III. NUMERICAL RESULTS

To fit the parameters α and β of our model, we consider recent results from numerical relativity in the range $q = 3 - 7$, $\chi_{\text{BH}} = 0 - 0.9$ and $C_{\text{NS}} = 0.13 - 0.18$. We neglect simulations at lower mass ratios, which are astrophysically less likely and cannot be modelled accurately by the simple formula assumed here. Larger spins and more compact stars would be interesting to consider, but have yet to be simulated. Indeed, according to Hebeler et al. [22], neutron stars of mass $M_{\text{NS}} \sim 1.4M_{\odot}$ could be in the range $C_{\text{NS}} = 0.15 - 0.22$. For the same neutron star mass, Steiner et al. [23] found the most likely compactness to be $C_{\text{NS}} = 0.17 - 0.19$.

We also limit the model to spins aligned with the orbital angular momentum and to low-eccentricity orbits. Misaligned spins have only been studied for one set of binary parameters [8], so that we do not have enough information about their influence on the disk mass to include them in the model. High-eccentricity mergers have been studied by East et al. [24, 25]. As for misaligned black hole spins, however, the data does not cover enough of the parameter space to be included in our fit. Additionally, eccentricity is only an issue for binaries formed in clusters: field binaries are expected to have negligible eccentricities at the time of merger. Finally, we neglect the influence of magnetic fields, as both Etienne et al. [15] and Chawla et al. [16] find their effect on the remnant mass to be small (except for large interior magnetic fields $B \gtrsim 10^{17}G$).

A list of all simulations used to fit our model is given in Table I. These results were obtained by three different groups: Kyoto [5] (SACRA code), UIUC [6] and the SXS collaboration [7, 8] (SpEC code). In those articles, the mass outside the black hole $M_{\text{NR}}^{\text{rem}}$ is measured at different times, which would introduce a bias in our fit. We choose to use the convention of Kyutoku et al. [5], where $M_{\text{NR}}^{\text{rem}}$ is measured 10ms after merger. For this reason, the values listed in Table I differ from the masses given in the tables of [6–8].

Only some of the simulations listed in Table I were published with explicit error measurements. There is thus some uncertainty on the accuracy of these results. From published convergence tests and our own experience with such simulations, we assume that a rough estimate for the numerical errors $\Delta M_{\text{NR}}^{\text{rem}}$ can be obtained by combining a 10% relative error and a 1% absolute error in the mass measurement, i.e.

$$\frac{\Delta M_{\text{NR}}^{\text{rem}}}{M_{\text{NS}}^b} = \sqrt{(0.1M_{\text{NR}}^{\text{rem}})^2 + 0.01^2}. \quad (9)$$

IV. PARAMETER ESTIMATES

A. Best-Fit parameters

We determine the parameters α and β of our model (Eq.6) through a least-square fit for the results of simulations 1-26 in Table I. Simulations 27-31, which do not lead to the formation of a disk, are not used directly — but we check that the model

TABLE I: Summary of the numerical results used. When more than one group simulated the same set of parameters, the average value is used. $\chi_{\text{BH}} = a_{\text{BH}}/M_{\text{BH}}$ is the dimensionless spin parameter of the black hole, $C_{\text{NS}} = M_{\text{NS}}/R_{\text{NS}}$ is the compactness of the star, $M_{\text{NR}}^{\text{rem}}$ is the remaining mass 10ms after merger (as measured in the numerical simulations), and M_{NS}^b is the baryon mass of the star.

ID	$\frac{M_{\text{BH}}}{M_{\text{NS}}}$	χ_{BH}	C_{NS}	$\frac{M_{\text{NR}}^{\text{rem}}}{M_{\text{NS}}^b}$	Code	Ref.
1	7	0.90	0.144	0.24	SpEC	[7]
2	7	0.70	0.144	0.05	SpEC	[7]
3	5	0.50	0.144	0.05	SpEC	[7]
4	3	0.90	0.144	0.35	SpEC	[8]
5	3	0.50	0.145	0.15	SpEC/SACRA	[5, 8]
6	3	0.00	0.144	0.04	UIUC/SPEC	[6, 8]
7	3	0.75	0.145	0.21	UIUC/SACRA	[5, 6]
8	5	0.75	0.131	0.25	SACRA	[5]
9	5	0.75	0.162	0.11	SACRA	[5]
10	5	0.75	0.172	0.06	SACRA	[5]
11	5	0.75	0.182	0.02	SACRA	[5]
12	4	0.75	0.131	0.25	SACRA	[5]
13	4	0.75	0.162	0.15	SACRA	[5]
14	4	0.75	0.172	0.12	SACRA	[5]
15	4	0.75	0.182	0.07	SACRA	[5]
16	4	0.50	0.131	0.19	SACRA	[5]
17	4	0.50	0.162	0.06	SACRA	[5]
18	4	0.50	0.172	0.02	SACRA	[5]
19	3	0.75	0.131	0.24	SACRA	[5]
20	3	0.75	0.162	0.16	SACRA	[5]
21	3	0.75	0.172	0.15	SACRA	[5]
22	3	0.75	0.182	0.10	SACRA	[5]
23	3	0.50	0.131	0.19	SACRA	[5]
24	3	0.50	0.162	0.11	SACRA	[5]
25	3	0.50	0.172	0.07	SACRA	[5]
26	3	0.50	0.182	0.03	SACRA	[5]
27	7	0.50	0.144	0.00	SpEC	[7]
28	3	-0.50	0.145	0.01	UIUC	[6]
29	5	0.00	0.145	0.01	UIUC	[6]
30	4	0.50	0.182	0.00	SACRA	[5]
31	3	-0.50	0.172	0.00	SACRA	[5]

is consistent with their results. We find

$$\alpha = 0.288 \pm 0.011 \quad (10)$$

$$\beta = 0.148 \pm 0.007, \quad (11)$$

for model $M_{\text{model}}^{\text{rem}}$ in which tidal forces are estimated from Newtonian physics, and

$$\tilde{\alpha} = 0.296 \pm 0.011 \quad (12)$$

$$\tilde{\beta} = 0.171 \pm 0.008 \quad (13)$$

for the modified model $\tilde{M}_{\text{model}}^{\text{rem}}$ in which the tidal forces are derived from the Kerr metric.

Error estimates are easier if we rewrite the models using singular value decomposition (see e.g p65-75 and p793-796 of Press et al. [26], and references therein), that is if we transform the basis functions of our model so that the parameters of the model have uncorrelated errors. For example, in the

case of the 'Newtonian' model we have

$$\begin{aligned} f_1 &= 0.851 (3q)^{1/3} (1 - 2C_{\text{NS}}) - 0.525 \frac{R_{\text{ISCO}}}{R_{\text{NS}}} \\ f_2 &= 0.525 (3q)^{1/3} (1 - 2C_{\text{NS}}) + 0.851 \frac{R_{\text{ISCO}}}{R_{\text{NS}}} \end{aligned}$$

$$\frac{M_{\text{model}}^{\text{rem}}}{M_{\text{NS}}^b} = Af_1 + Bf_2. \quad (14)$$

The best-fit parameters A and B are then

$$A = 0.323 \pm 0.013 \quad (15)$$

$$B = 0.026 \pm 0.001, \quad (16)$$

where the errors on A and B are independent (while the errors on α and β were strongly correlated).

B. Goodness-of-fit

The ability of these models to fit the numerical results within their errors $\Delta M_{\text{NR}}^{\text{rem}}$ can be estimated through the reduced χ^2

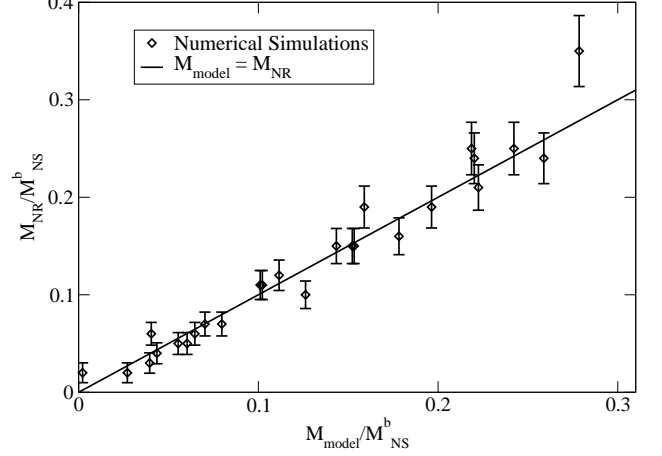
$$\chi^2 = \frac{1}{N_{\text{df}}} \sum_{i=1}^{26} \left(\frac{M_{\text{model}}^{\text{rem},i} - M_{\text{NR}}^{\text{rem},i}}{\Delta M_{\text{NR}}^{\text{rem},i}} \right)^2 \quad (17)$$

where $N_{\text{df}} = 26 - N_{\text{params}} = 24$ is the number of degrees of freedom, and the index i refers to the ID of the numerical simulations. The 'Newtonian' model $M_{\text{model}}^{\text{rem}}$ and the 'Kerr' model $\tilde{M}_{\text{model}}^{\text{rem}}$ are equally good fit to the data, with $\chi^2 = 0.98$ and $\chi^2 = 0.96$ respectively. By comparison, the best-fit results for model M_{model}^0 (in which we do not correct d_{tidal} by the factor $[1 - 2C_{\text{NS}}]$) has a much larger $\chi^2 = 4.04$. Adding a constant term γ to either $M_{\text{model}}^{\text{rem}}$ or $\tilde{M}_{\text{model}}^{\text{rem}}$ leads to $\chi^2 = 1.00$.

A comparison between the simple model $M_{\text{model}}^{\text{rem}}$ and the numerical results is shown in Fig. 1, in which we plot $M_{\text{NR}}^{\text{rem}}$ as a function of $M_{\text{model}}^{\text{rem}}$ for simulations 1-26. We can see that the difference between the modelled and measured masses is generally smaller than the errors expected from Eq. 9. The main exception is the large remnant mass observed in case 4. We suspect that our model, which assumes that the remnant mass scales linearly with R_{ISCO} and d_{tidal} , breaks down for remnant masses greater than about 20 – 25% of the neutron star mass. A non-linear relation between these distances and the remnant mass might perform better in that regime, but more numerical simulations are required to test that hypothesis.

The more complex model $\tilde{M}_{\text{model}}^{\text{rem}}$ offers very similar results: for cases 1-26, the worst disagreement between the models is $0.008 M_{\text{NS}}^b$ (for case 3) while their rms difference is $0.004 M_{\text{NS}}^b$.

FIG. 1: Predictions of the best-fit model (diamonds) for simulations 1-26. The solid line represents the ideal $M_{\text{model}}^{\text{rem}} = M_{\text{NR}}^{\text{rem}}$ result, while the error bars correspond to the estimated numerical errors $\Delta M_{\text{NR}}^{\text{rem}}$.



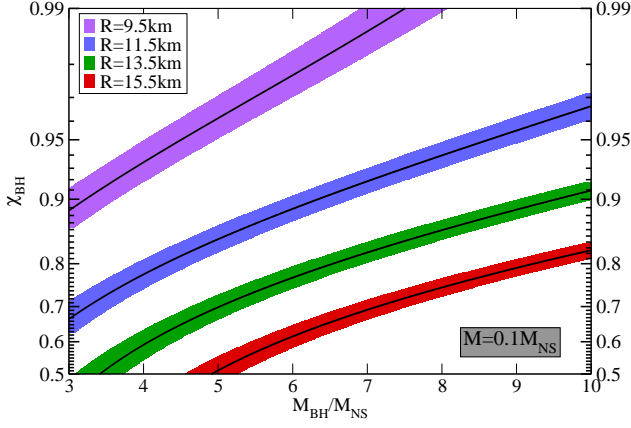
C. Error Estimates

Estimating the error in the mass predictions of our model from the statistical errors in the parameters α and β is likely to be misleading. Differences between the numerical results $M_{\text{NR}}^{\text{rem}}$ and the predictions of the model $M_{\text{model}}^{\text{rem}}$ come from multiple sources: the numerical error $\Delta M_{\text{NR}}^{\text{rem}}$ of course, but also a physical spread of the exact mass remnants around the predictions of the model. A part of that spread at least should be due to differences in the outcome of BHNS mergers for binaries with the same parameters ($M_{\text{BH}}, M_{\text{NS}}, \chi_{\text{BH}}, C_{\text{NS}}$), but different equations of state (i.e. neutron stars with the same radius but a different internal structure). But more generally, it is unlikely that the simple equations used here can perfectly represent the complex dynamics of a BHNS merger.

From the fact that we measured $\chi^2 \sim 1$, we know that the errors $M_{\text{model}}^{\text{rem}} - M_{\text{NR}}^{\text{rem}}$ are compatible with a gaussian distribution of variance $\Delta M_{\text{NR}}^{\text{rem}}$. This is already indicative of the likely existence of a non-zero physical spread around the results of the model. The estimated numerical errors $\Delta M_{\text{NR}}^{\text{rem}}$ are indeed more of an upper-bound on the errors in the simulations than the width of an expected gaussian distribution. In the absence of a difference between the real physical outcome of a merger and the output of the model, we would thus expect χ^2 to be lower than 1. How much of the measured errors $M_{\text{model}}^{\text{rem}} - M_{\text{NR}}^{\text{rem}}$ comes from numerical errors and how much from actual differences between the model and the physical reality is hard to determine, especially considering that the numerical errors are not well known. A more cautious approach to estimate the uncertainty in the model is thus to consider $\Delta M_{\text{NR}}^{\text{rem}}$ as a conservative upper bound on the variance of the error in the model itself.

Fig. 2 shows contours of $M_{\text{model}}^{\text{rem}} = 0.1 M_{\text{NS}}^b$ for various neutron star compactness. The general features of this plot are not surprising: the formation of massive disks is known to be favored by low mass ratios, high black hole spins and large neutron stars. But our model allows for the determination of

FIG. 2: $M_{\text{model}}^{\text{rem}} = 0.1M_{\text{NS}}^b$ contours for, from top to bottom, neutron star compactness $C_{\text{NS}} = 0.22, 0.18, 0.155, 0.135$ (i.e. $R_{\text{NS}} \approx 9.5, 11.5, 13.5, 15.5\text{km}$ for $M_{\text{NS}} = 1.4M_{\odot}$). For each compactness, we have $M_{\text{model}}^{\text{rem}} > 0.1M_{\text{NS}}^b$ above the plotted contour. The shaded regions encompass the portions of phase space for which $M_{\text{model}}^{\text{rem}} = 0.1M_{\text{NS}}^b \pm \Delta M_{\text{NR}}^{\text{rem}}$. SGRBs are extremely unlikely to occur below the green region ($C_{\text{NS}} = 0.155$). Note that the scale is chosen in order to zoom on the high-spin region.



the region of parameter space in which a certain amount of matter will remain available at late times with fairly high accuracy: at least within the spread $\Delta M_{\text{NR}}^{\text{rem}} \approx 0.02M_{\odot}$ or, if we consider a measurement of M^{rem} as a way to determine the radius of a neutron star, within $\Delta R_{\text{NS}} \lesssim 0.5\text{km}$.

Another important issue is the validity of the model outside the parameter range currently covered by numerical relativity. It is indeed possible that larger errors will be found for more compact neutron stars ($C_{\text{NS}} > 0.18$) or larger mass ratios ($M_{\text{BH}} > 7M_{\text{NS}}$). However, given that our model fits the numerical data over a fairly wide range of parameters, and is derived from the physics of tidal disruption, it is likely to give decent results over most of the astrophysically relevant parameter space — with the notable exception of configurations leading to very large remnant masses $M^{\text{rem}} \gtrsim 0.20 - 0.25M_{\text{NS}}$ (i.e. for nearly-extremal black hole spins and low mass ratios), and probably of the asymptotic regime $\chi_{\text{BH}} \rightarrow 1$ where scalings valid in the range $\chi = 0 - 0.9$ might break down.

V. RESULTS

The models described in the previous sections can be used to easily approximate the region of parameter space in which disruption occurs, or in which a certain amount of mass will remain available at late times. Such predictions are particularly important when trying to determine which BHNS mergers are likely to lead to short gamma-ray bursts (SGRBs). BHNS mergers ending with the formation of a massive accretion disk might power SGRBs. In the absence of a disk, on the other hand, the only observational signatures of BHNS mergers are their gravitational wave emissions, as well as potential electromagnetic or neutrino precursors (see e.g. Tsang

et al. [27]).

The minimum remnant mass required to get SGRBs is currently unknown, and is likely to vary across the parameter space: the fraction of the remnant mass which, at any given time, is in a long-lived accretion disk around the black hole (as opposed to the tidal tail or unbound ejecta) is by no means a constant, nor are the physical characteristics of that disk. We know, for example, that at high mass ratios a larger fraction of the mass is initially in an extended tidal tail than for lower mass black holes [7]. Nevertheless, $M_{\text{model}}^{\text{rem}}$ is already a useful prediction, providing a good estimate of the amount of material available for post-merger evolution. Additionally, any configuration for which $M_{\text{model}}^{\text{rem}} = 0$ can be immediately rejected as a potential SGRB progenitor.

Predictions for the mass of neutron star material remaining outside the black hole at late times are detailed in Figs. 3-5, in which we plot contours of the remnant mass as a function of the mass ratio and black hole spin. Each figure correspond to a different neutron star compactness, covering the range of radii expected from the theoretical results of Hebeler et al. [22]. Experimental measurements of neutron star radii are still fairly difficult, but studies of bursting X-ray binaries by Özel et al. [28–30] tend to favor the lower range of potential radii ($R_{\text{NS}} \approx 9 - 12\text{km}$). Steiner et al. [23], after reassessing the errors in the measurement of neutron star radii from X-ray bursts, derived a parametrized equation of state which takes into account both the astrophysical measurements and results from nuclear theory. They predict that $R_{\text{NS}} \approx 11 - 12\text{km}$ for $M_{\text{NS}} = 1.4M_{\odot}$. We can thus consider Fig. 3 and Fig. 5 as bounding the range of potential neutron star radii, while Fig. 4 is around the most likely neutron star size.

The strong dependence of the remnant mass in the radius of the star is particularly noteworthy. In the most likely astrophysical range of mass ratios ($q \sim 5 - 10$), remnant masses $M^{\text{rem}} = 0.1M_{\text{NS}}$ can be achieved for moderate black hole spins $\chi_{\text{BH}} \approx 0.7 - 0.9$ if we consider neutron stars with $C_{\text{NS}} = 0.155$ ($R_{\text{NS}} \approx 13.5\text{km}$), as in Fig. 3. But at the other end of the range of potential neutron star radii, for $C_{\text{NS}} = 0.22$ ($R_{\text{NS}} \approx 9.5\text{km}$), the much more restrictive condition $\chi_{\text{BH}} \approx 0.9 - 0.999$ applies (Fig. 5). For a neutron star in the range of compactness favored by Steiner et al. [23] ($C_{\text{NS}} = 0.18$, or $R_{\text{NS}} \approx 11.5\text{km}$), keeping 10% of the neutron star material outside the black hole requires spins $\chi_{\text{BH}} \approx 0.8 - 0.97$, an already fairly restrictive condition (Fig. 4).

This naturally implies that the rate of SGRBs produced as a result of BHNS mergers is very sensitive to the equation of state of nuclear matter, and in particular to the size of neutron stars. Determining that rate is unfortunately impossible without knowledge of the number of BHNS mergers, and of the distributions of black hole spins and mass ratios. Additionally, M^{rem} is not a fully reliable predictor of the ability of a given BHNS binary to power a SGRB. Nonetheless, the importance of the equation of state can be fairly easily understood by simply computing the area of the region above a certain contour of $M_{\text{model}}^{\text{rem}}$ for various values of C_{NS} . Let us define $\chi^c(M, C_{\text{NS}}, q)$ as the critical spin above which $M_{\text{model}}^{\text{rem}} > M$

FIG. 3: Contours $M_{\text{model}}^{\text{rem}} = (0, 0.05, 0.1, 0.15, 0.2)M_{\text{NS}}^b$ for a star of compactness $C_{\text{NS}} = 0.155$ ($R_{\text{NS}} \approx 13.5\text{km}$ for $M_{\text{NS}} = 1.4M_{\odot}$). The shaded regions correspond to portions of parameter space in which no matter remains around the black hole (bottom/red), more than $0.2M_{\text{NS}}^b$ remains and massive disks should be the norm (top/green), and an intermediate region in which lower mass disks will form (center/blue). Note that the scale is chosen in order to zoom on the high-spin region.

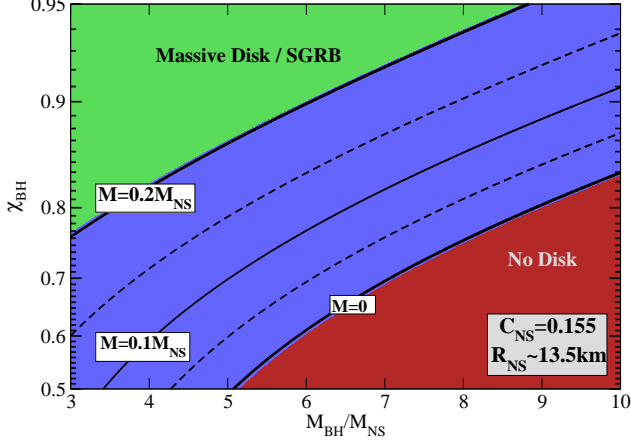


FIG. 4: Same as Fig. 3, but for $C_{\text{NS}} = 0.18$ ($R_{\text{NS}} \approx 11.5\text{km}$).

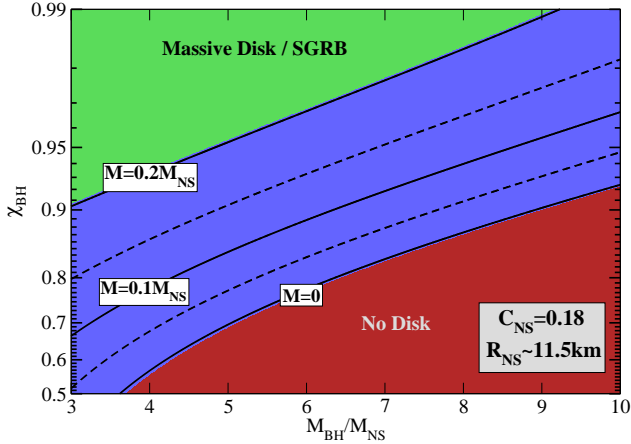


FIG. 5: Same as Fig. 3, but for $C_{\text{NS}} = 0.22$ ($R_{\text{NS}} \approx 9.5\text{km}$).

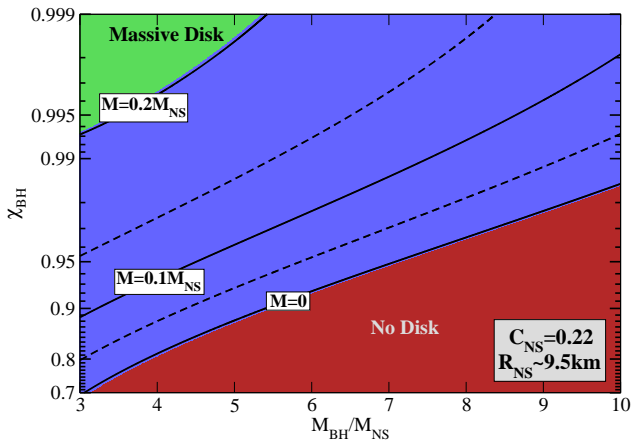


TABLE II: Fraction $\phi(M, C_{\text{NS}})$ of the parameter space within $q = 5 - 10$, $\chi_{\text{BH}} = 0 - 1$ for which $M_{\text{model}}^{\text{rem}} > M$ for various neutron star compactness C_{NS} and critical masses M .

C_{NS}	$\phi(0, C_{\text{NS}})$	$\phi(0.1M_{\text{NS}}^b, C_{\text{NS}})$	$\phi(0.2M_{\text{NS}}^b, C_{\text{NS}})$
0.135	0.46	0.30	0.16
0.155	0.29	0.17	0.07
0.180	0.16	0.08	0.02
0.220	0.05	0.01	0.00

and

$$\phi(M, C_{\text{NS}}) = \frac{\int_5^{10} [1 - \chi^c(M, C_{\text{NS}}, q)] dq}{5}. \quad (18)$$

Then, $\phi(M, C_{\text{NS}})$ represents the fraction of binaries with mass remnants greater than M assuming that the distributions of mass ratios and spins are uniform within the $q = 5 - 10$ and $\chi_{\text{BH}} = 0 - 1$ range respectively. As we decrease the size of the neutron star from $C_{\text{NS}} = 0.155$ to $C_{\text{NS}} = 0.22$, Table II shows that we go from about 20% of the parameter space in which significant disks are possible to about 1%! This does not necessarily mean that SGRBs are impossible for $C_{\text{NS}} \sim 0.22$ — but certainly indicate that they would occur in a non-negligible fraction of BHNS mergers only if quasi-extremal spins are the norm.

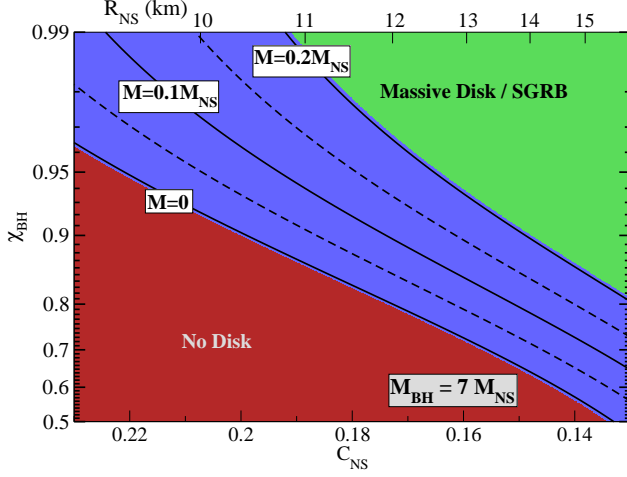
These predictions can be compared with previous estimates for the disk mass obtained by Pannarale et al. [19]. In [19], an estimate of the mass available at late times is obtained through the use of a toy-model studying the tidal forces acting on the neutron star, which is represented by a tri-axial ellipsoid. If the qualitative features of both models are fairly similar, we find that the use of more recent numerical results (and, in particular, the availability of mass estimates for larger mass ratios) causes significant differences in their predictions. Specifically, the results presented here indicate that it is a lot more difficult to create massive accretion disks at high mass ratios than what that previous model indicated. On the other hand, at low mass ratios $q \sim 3$ our model predicts significantly higher final masses — probably because many of the numerical simulations used to calibrate [19] are now known to have underestimated the amount of matter remaining outside the black hole. As opposed to [19], our model is unlikely to capture the behavior of BHNS mergers with $q \sim 1$, when finite-size effects begin to make it more difficult to form massive disks.

We can also revisit the condition derived by Taniguchi et al. [18] for the parameters allowing disk formation in the case of non-spinning black holes. Requiring $M_{\text{model}}^{\text{rem}} > 0$ for $\chi_{\text{BH}} = 0$ is equivalent to imposing an upper bound on the neutron star compactness,

$$C_{\text{NS}}^{\chi_{\text{BH}}=0} \lesssim \frac{1}{2 + 2.14q^{2/3}}. \quad (19)$$

For massive black holes ($q \gtrsim 7$), this result is very similar to what Taniguchi et al. [18] found by studying quasi-equilibrium sequences of BHNS binaries. At low mass ratio, our results are less favorable to tidal disruption and disk formation. This is in agreement with the numerical simulations

FIG. 6: Contours of $M_{\text{model}}^{\text{rem}}$ for binaries with mass ratio $M_{\text{BH}} = 7M_{\text{NS}}$ ($M_{\text{BH}} \approx 10M_{\odot}$). Shown are contours for $M_{\text{model}}^{\text{rem}} = (0, 0.05, 0.1, 0.15, 0.2)M_{\text{NS}}^b$. The shaded regions are as in Fig. 3 and the neutron star radius (top scale) is computed assuming a star of ADM mass $M_{\text{NS}} = 1.4M_{\odot}$.



performed by Kyutoku et al. [9], even though the results for low mass ratio, non-spinning BHNS mergers published in [9] were not taken into account when fitting our model.

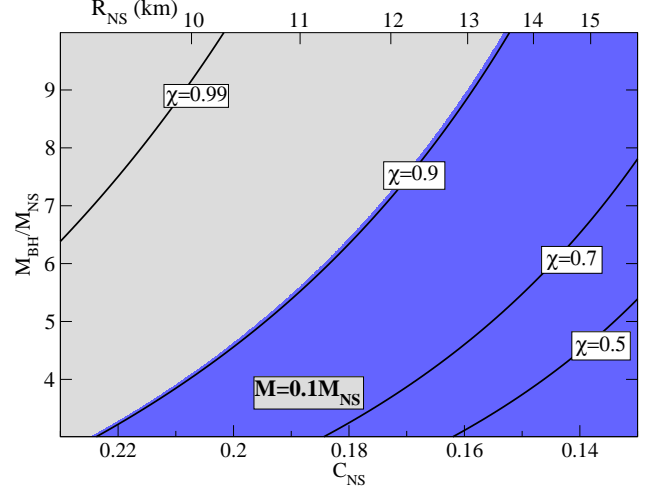
Current population synthesis models estimate the peak of the distribution of black hole masses in BHNS systems to be around $M_{\text{BH}} \sim 10M_{\odot}$, or $M_{\text{BH}} \sim 7M_{\text{NS}}$ [13, 14]. Fig. 6 offers clearer information on the behavior of BHNS systems in that regime. We see that no disk can form for $\chi_{\text{BH}} < 0.9$ unless $R_{\text{NS}} > 10.5\text{km}$. That condition becomes $R_{\text{NS}} > 12\text{km}$ if we require at least $0.1M_{\text{NS}}^b$ outside the black hole at late times. Results for BHNS binaries with higher black hole spins ($\chi_{\text{BH}} \rightarrow 1$) should of course be considered with caution: indeed, no mergers of BHNS binaries have been published for $\chi_{\text{BH}} > 0.9$ or $C_{\text{NS}} > 0.18$, and such simulations would be required to rigorously test the accuracy of these predictions in extreme regions of the parameter space. Nonetheless, our model indicates that quasi-extremal spins are at least a necessary condition for the formation of massive disks for $M_{\text{BH}} \approx 10M_{\odot}$ and $R_{\text{NS}} \leq 12\text{km}$.

The minimum spin requirement for massive disk formation across the parameter space of BHNS binaries is shown in Fig. 7, in which the black hole spin needed to keep 10% of the neutron star mass outside the black hole at late times is plotted. Figs. 6 and 7 both indicate the existence of an extended region of parameter space ($C_{\text{NS}} \sim 0.18 - 0.22$, $\chi_{\text{BH}} \sim 0.9 - 1$) which is likely to be astrophysically relevant but remains numerically unexplored, and in which the outcome of BHNS mergers varies significantly.

VI. CONCLUSIONS

We constructed a simple model predicting the amount of matter remaining outside the black hole about 10ms after a BHNS merger, based on comparisons between the binary sep-

FIG. 7: Contours $M_{\text{model}}^{\text{rem}} = 0.1M_{\text{NS}}^b$ for varying black hole spins $\chi_{\text{BH}} = (0.5, 0.7, 0.9, 0.99)$. The grey region contains spins above the maximum value reached by numerical simulations.



aration at which the neutron star is expected to be disrupted by tidal forces from the black hole and the radius of the innermost stable circular orbit around the hole. We show that the model can reproduce the results of recent general relativistic simulations of non-precessing, low-eccentricity BHNS mergers within a few percents of the total mass of the neutron star. The simplest best-fit model is

$$\frac{M_{\text{model}}^{\text{rem}}}{M_{\text{NS}}^b} \approx 0.288 \left(3 \frac{M_{\text{BH}}}{M_{\text{NS}}} \right)^{1/3} \left(1 - 2 \frac{M_{\text{NS}}}{R_{\text{NS}}} \right) - 0.148 \frac{R_{\text{ISCO}}}{R_{\text{NS}}}$$

(in units in which $G = c = 1$).

These mass predictions should be valid at the very least within the range of parameters currently covered by numerical simulations ($M_{\text{BH}} = 3 - 7M_{\text{NS}}$, $R_{\text{NS}} = 11 - 16\text{ km}$, $a_{\text{BH}}/M_{\text{BH}} = 0 - 0.9$), and are likely to remain fairly accurate within most of the astrophysically relevant parameter space.

Using this model, it becomes easy to estimate the region of parameter space in which large amounts of matter remain outside the black hole for long periods of time. This is of particular importance when studying whether BHNS mergers can result in short gamma-ray bursts. Our results show the strong dependence of the remnant mass in the radius of the neutron star: whether the equation of state of neutron stars is at the soft or stiff end of its potential range could easily translate into an order of magnitude difference in the rate of gamma-ray bursts originating from BHNS mergers. It is also quite clear that high black hole spins are likely to be a prerequisite for the formation of massive disks. Neutron stars in the middle of the theoretically allowed range of radii ($R_{\text{NS}} \sim 11.5\text{km}$) require spins $a_{\text{BH}}/M_{\text{BH}} \gtrsim 0.8$ for about 10% of the neutron star material to remain outside the hole, while quasi-extremal spins are necessary for the most compact stars.

The validity of our model is currently limited to black hole spins aligned with the orbital angular momentum and remnants below $\sim 20 - 25\%$ of the neutron star mass, due to the lack of numerical data available for precessing binaries and

high mass remnants. Extending the model to cover these interesting parts of the parameter space would certainly be useful, but would require a large number of computationally intensive simulations to be performed (particularly to cover misaligned black hole spins). A few additional simulations using high mass ratios or small neutron star radii together with relatively large spins ($\chi_{\text{BH}} \gtrsim 0.9$) would also be extremely helpful, allowing better estimates of the errors in the model for binary parameters which are astrophysically relevant but have never been considered by numerical relativists.

The extreme simplicity of these models should make them useful tools to obtain cheap but reliable estimates of the results of BHNS mergers across most of the astrophysically relevant parameter space, as well as to help determining which numer-

ical simulations to perform in order to study given physical effects.

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